

mien-Schlichting type from vortices, and that is a phase-reversal of the longitudinal velocity fluctuation across the critical layer for the vortex. But Hama³ does not consider it to be sufficient, nor even necessary. Also, Michalke's calculations² of wavy disturbances in a free shear layer show that the existence of a phase reversal at the critical layer depends on whether the wave is neutral or amplifying, and whether its amplification is temporal or spatial. Further, even for the Tollmien-Schlichting wave in a Blasius profile, the phase reversal criterion is vague. In application, experimentalists have more or less implicitly looked for a phase reversal about the critical layer as the wave passed directly over a hot-wire location. But streamline calculations of a Tollmien-Schlichting wave (Fig. 16.14 of Ref. 9) actually do show a phase reversal about the critical layer, for the velocity profile at $\frac{1}{2}$ wavelength ahead of the wave center. The extent of this phase change will depend on the eigenfunction solution, and is thus dependent on frequency, Reynolds number and the shear-layer profile. Thus, the phase-reversal criterion also appears inadequate.

As for Michalke's characterization of a vorticity extrema as a vortex, consider Uchida's solution⁹ for oscillating pipe flow. Here, maxima of vorticity appear periodically, yet all streamlines are at all times identically straight and parallel. It therefore appears that Michalke's criterion is also inadequate.

Granted the lack of a completely adequate objective distinction between a wave and a vortex, the fact remains that various investigators do distinguish subjectively. We will now hypothesize some kind of consensus about particular cases and consider the applicability of Michalke's comments¹ to our work.¹⁶

First, with regard to the identification of a discrete vortex by the roll-up of a smoke line, we note that Hama³ repeated his calculations for a u component of velocity identically equal to zero, and still obtained the characteristic roll-up of the smoke lines. There is no phase reversal of u in this case, only a fluctuating v component. It is clear that a Tollmien-Schlichting wave would also cause the smoke lines to roll up, and, as we have said, customary usage has been to not call these waves vortices.

But suppose we now adopt Michalke's view of a maximum in vorticity being legitimately characterized as a discrete vortex. Then Michalke has argued² that since a (smoke) streakline is approximately a line of constant vorticity, that a smoke line roll-up constitutes a positive identification of a vorticity maximum (or concentration, in Michalke's sense). But streaklines are constant vorticity lines only in inviscid flow. In some regions of even high Reynolds number flows, significant viscous production of vorticity may occur. That Michalke's approximation is not entirely valid for a wall boundary layer may be easily demonstrated. If vorticity production is negligible, then the maximum vorticity in the waves can nowhere exceed the maximum vorticity in the undisturbed shear layer²; but Kovasznay et al.⁷ have experimentally measured maximum vorticity equal to twice the maximum in the undisturbed Blasius profile. It is clear then that our original claim¹⁰ stands unaltered; the roll-up of a streakline cannot constitute a positive identification of discrete vortices, and the interpretation of region R_1 is not possible from the smoke technique alone.

Finally, we consider another hypothetical consensus. Suppose that it is agreed that R_1 is a region of Tollmien-Schlichting waves, which all parties agree not to call vortices. Suppose also that all parties consider the characteristics of Michalke's spatially growing disturbances in a free shear layer,² noting the phase reversal and the vorticity extrema, and agree with Michalke that these may be legitimately characterized as discrete vortices. What is the relation to our work?¹⁰ Michalke's discrete vortices develop in a free shear-layer *inflectional* profile, not a Blasius-type profile. It is well known^{6,7,4} that such an instantaneous in-

flexional profile does develop in region R_2 , and that this instantaneous inflexional profile develops a *secondary* instability, which ultimately cascades⁴ into region R_3 —a region that we and all other investigators have characterized as *discrete vortices*.

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Correction to Tsien's Listing of Ideal Gas Flow

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IN his article¹ on gasdynamics, Tsien states that there are eight consistent types of frictionless, adiabatic flows for an ideal gas in the absence of body forces, and he tabulates the flows. However, careful examination shows that two of the flows have identical classifications (4 and 8 in Tsien's table). This raises the question of whether there is an error in the table, or whether there are, in fact, only seven consistent flows. A correct listing of fluid motion of the type being considered here is not available in the literature.

By using the energy equation with stagnation enthalpy as an independent variable, Crocco's theorem in both the steady and nonsteady form, and Eqs. (7-21) from Tsien's article, one can establish criteria for flow consistency. There are 16 possible adiabatic, frictionless flows with no body forces, nine of which are inconsistent with the equations of motion. In fact, only seven classifications are consistent with the equations of motion, not eight. Flow number 8 in Tsien's table should be deleted.

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